Comparison of Some Selected Multiaxial Fatigue Assessment Criteria

Adam Niesłony[a], Cetin Morris Sonsino[b]

[a] Opole University of Technology, Opole, Poland
[b] Fraunhofer Institute for Structural Durability and System Reliability LBF in Darmstadt, Germany

Abstract In this paper, experimental data of constant amplitude fatigue tests of notched specimens of two ductile steels obtained by Simbürger and by Atzori et al. are taken as the basis for an evaluation by five fatigue assessment criteria. These hypotheses were Dang Van, Equivalent Effective Shear, two Energy Criteria and Gough-Pollard. The multiaxial loading state was achieved by the combination of axial or bending and torsion in in-phase and out-of-phase cases with constant local stress ratios $\tau_{xya} / \sigma_{xa} = 0.58$ and 0.53, respectively. The reason that data of notched specimens were evaluated is given by the fact that components always have stress concentrations and that their behaviour can be best assessed by notched members. Using selected fatigue assessment criteria Woehler curves were calculated and compared with those obtained experimentally. From the results obtained, it was concluded that the best results were obtained by the integral hypothesis of Equivalent Effective Shear, which is based on plane oriented shear stresses.

1 INTRODUCTION

The driving force for various international research activities in the field of multiaxial fatigue lies in the fact that the conventional hypotheses, like those based on the change of strain energy of distortion or on the maximum value of shear stresses, which were formulated for static loading, give non-acceptable results during evaluation of fatigue strength when the direction of principal stresses changes, especially for ductile materials. In the meantime, agreement has been reached that, for the solution of the problem, local concepts based on stress, strain or energy parameters must be used, since, for failure, local loading quantities are responsible. It is well known that these concepts can be flexibly extended to consider, for example, the influence of stress gradients or surface roughness. Here, three kinds of models can be established: principal stresses referred models, critical plane referred models and integral models which can proceed from the critical plane concept. The selection of the model for fatigue life assessment depends particularly on the type of material and especially on its ductility [1].

The aim of this paper is to compare five selected well-known multiaxial fatigue life criteria, which have different theoretical backgrounds. The quality of fatigue life assessment will be established by comparison of computed Woehler curves with experimental data obtained for two ductile steels with notched specimens. The reason that data for notched specimens were evaluated is given by the fact that components always have stress concentrations and that their behaviour can be best assessed by notched members. Data obtained with multiaxially loaded unnotched specimens of ductile steels render incorrect results, if deformation conditions in the notch due to stress gradients are not considered [2].

2 EXPERIMENTAL DATA

For the purpose of verification, experimental data, which can be found in the literature, were used. While searching for suitable data, special attention was paid to the kind of experimental tests and their complexity. The following assumptions have been made concerning the experimental data in order to perform the comparison of selected multiaxial fatigue criteria for ductile materials correctly:

- The experimental data should be obtained under constant amplitude cyclic loading on smooth and notched specimens,
- the multiaxial stress state is realised by combination of axial (or bending) loading with torsion,
- the loading components are equal in frequency without and with phase shift ($\varphi = 0°$ and $\varphi = 90°$),
- experimental Woehler curves are presented using local stresses.

Based on these reasons, two sets of experimental results were selected, which have been presented by Simbürger in [3] and by Atzori et al. in [4]. Details of the particular tests are described in the following subsections.

2.1 Data presented by Simbürger

The fatigue data obtained by Simbürger [3] are well-known and often used for verification of multiaxial fatigue assessment criteria [5, 6, 7]. The testing program comprised plain and notched specimens made of Ck 45 steel (SAE 1045). The tests were load controlled. A constant amplitude loading was used with equal frequency of stress tensor components to perform several tests with various mean values and phase shifts. The results obtained were presented as Woehler curves using values of local stress amplitudes

$$\sigma_a = K_t \sigma_{a,nom} \,. \tag{1}$$

The criterion for fatigue failure was the crack initiation defined as crack depth of 0.25 mm. Crack depth was monitored and detected during the tests by the ultrasonic method. Uniaxial tests for determining only material data were performed on unnotched plain and hollow specimens. Tests under bending, torsion and combined loading were performed on notched specimens with 5 mm notch radius, having the theoretical stress concentration factor K_{tt} = 1.24 under torsion and K_{tb} = 1.49 under bending.

2.2 Data presented by Atzori et al.

In [4], Atzori et al. presented fatigue strength data of V-notched specimens with notch radius equal to 0.5 mm made of the ductile C40 steel (SAE 1040) in the normalised state, subjected to combined tension and torsion loading. The theoretical stress concentration factor for notched specimens were equal to K_{ta} = 3.68 under axial loading, and K_{tt} = 1.95 under torsion. Experimental data are presented as Woehler curves first using nominal stresses, where N_r means number of cycle to total rupture of the specimen. The fatigue life to crack initiation N_{cr} was unfortunately not determined.

3 MULTIAXIAL FATIGUE ASSESSMENT CRITERIA

3.1 Dang Van criterion

In this criterion, a volume of material is investigated under the influence of a given stress state at any time *t*. However, in the evaluation of the criterion, the microscopic scale of the order of grain size is used, which is a subdivision of the macroscopic scale usually considered by engineers. In this scale, local plasticity may take place within some favourably oriented grains without influence on the macroscopic deformation, which can be still elastic. According to Dang Van et al. [8, 9], the crack initiation will happen in critically oriented grains of material volume, which have undergone plastic strains, if, for at least one cycle

$$f_{DV}\left(\sigma_{ij}(t)\right) \geq 0 \tag{2}$$

is obtained, where the f_{DV} function is some appropriate function of the local stress state. A simple form of the criterion has been proposed

$$f_{DV}\left(\sigma_{ij}(t)\right) = \tau(t) \pm a_{DV}\, p_h(t) \mp b_{DV} \tag{3}$$

as a linear relationship between two parameters. The first one is the shear stress determined according to the Tresca criterion

$$\tau(t) = \tau_T(t) = \frac{1}{2}\left(\sigma_1 - \sigma_3\right). \tag{4}$$

The second one is the hydrostatic tension

$$p_h = \frac{1}{3}\left(\sigma_1 + \sigma_2 + \sigma_3\right), \tag{5}$$

which is an important parameter with regard to crack opening. Two material constants a_{DV} and b_{DV} can be evaluated using experimentally obtained fatigue characteristic under simple loading conditions. Assuming that the fatigue effort function (3) for a given number of cycles $N_f = N$ reaches the critical value zero, the linear equation of the form

$$b_{DV} = \tau - a_{DV}\, p_h \tag{6}$$

is obtained, which represents the so called Dang Van line. Two characteristic points P_1 and P_2 can be found on the Dang Van line considering the case of combined axial loading or bending with torsion. The first one is located on the shear stress axis and its value is equal to the fatigue strength for torsion at a given number of cycles

$$P_1\left(p_h = 0, \tau = \tau_{aN}\right) \quad \text{for} \quad N_f = N . \tag{7}$$

The second one must be obtained from the Woehler-curve for axial loading

$$P_2\left(p_h = p_h(\sigma_{aN}), \tau = \tau_T(\sigma_{aN})\right) \quad \text{for} \quad N_f = N . \tag{8}$$

Normally, the Dang Van criterion is applied using material data obtained with uniaxially loaded unnotched specimens. However, as these data do not contain the influence of stress gradients, in this study the criterion was applied also with data determined with uniaxially loaded notched specimens. The coordinates of the point P_2 can be determined as follows:

$$\tau_T(\sigma_{aN}) = \frac{\sigma_{aN}}{2} \tag{9}$$

$$p_h(\sigma_{aN}) = \frac{\sigma_{aN}}{3}, \qquad (K_{ta} = 1.0); \qquad p_h(\sigma_{aN}) = \frac{1.215\sigma_{aN}}{3}, \ (K_{tb} = 1.49) \tag{10}$$

and the two material parameters a_{DV} and b_{DV}

$$b_{DV} = \tau_{aN} , \tag{11}$$

$$a_{DV} = \frac{3(\sigma_{aN} - 2\tau_{aN})}{2\sigma_{aN}} \quad (K_{ta} = 1.0); \qquad a_{DV} = \frac{3(\sigma_{aN} - 2\tau_{aN})}{2 \cdot 1.215\sigma_{aN}} \quad (K_{tb} = 1.49). \tag{12}$$

3.2 Effective Equivalent Stress Hypothesis (EESH)

The EESH [10] assumes that the failure of ductile materials under multiaxial stress state is initiated by an equivalent stress calculated according to the modified von Mises criterion in following form

$$\sigma_{eqa}(\delta_{xy} = 0°) = \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa}\sigma_{ya} + f_G 3\tau_{xya}^2} , \tag{13}$$

with

$$f_G = \frac{\sigma_{eq\,\text{(pure axial or bending load)}}}{\sigma_{eq\,\text{(pure torsion)}}} = \frac{\sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa}\sigma_{ya}}}{\sqrt{3}\tau_{xya}} . \tag{14}$$

The f_G parameter considers, for the shear stress component in equation (13), the size effect due to the different stress gradients resulting from bending and torsion. Generally, when the slopes for fatigue curves for bending end torsion are different, the parameter f_G depends on the number of cycles N_f. Then the coefficient f_G can be calculated for every number of cycles $N_f = N$ or for a single value of selected number of cycles $N_f = N_k$.

The interaction of shear stresses in various interference planes φ, in particular in the case of time-variable principal stress directions, which initiates corresponding dislocations, is taken into account by determining an effective shear stress

$$\tau_{arith} = \frac{1}{\pi}\int_0^{\pi} \max_t \left|\tau_n(t,\varphi)\right| d\varphi . \tag{15}$$

The effective shear stress is then used for determination of the effective equivalent stress

$$\sigma_{eqa} = \sigma_{eqa}(\varphi = 0°)\frac{\tau_{arith}}{\tau_{arith}(\delta = 0°)}\sqrt{G^Z} \tag{16}$$

with

$$G = \frac{1 + K_{ta}}{1 + K_{tt}} \ \text{or} \ G = \frac{1 + K_{tb}}{1 + K_{tt}}, \qquad Z = 1 - \left(\frac{\delta - 90°}{90°}\right)^2 . \tag{17}$$

The root in equation (16) considers the influence of the material volume affected by the rotating principal stress axes on the magnitude of the effective equivalent strain in the case of a phase displacement. The ratio G can be computed with corresponding stress concentration factors.

3.3 Energy based criterion proposed by Lagoda et al.

Lagoda et al. [11] formulated a criterion, which is a generalization of some energy criteria of multiaxial fatigue effort [15], where the normal strain energy $W_n(t)$ and the shear strain energy $W_{ns}(t)$ densities in the critical plane are used to establish a parameter which is responsible for fatigue damage. Detailed assumptions of the proposed criterion can be found in [11]. The critical plane position was determined by the maximum shear strain energy density, what lead to:

$$\max_t \{b_L W_{ns}(t)\} + a_L W_n(t) = f_L(N) \,. \tag{18}$$

From (18) the following equation for the equivalent parameter of strain energy density is derived:

$$W_{eq}(t) = \max_t \{b_L W_{ns}(t)\} + a_L W_n(t) \,. \tag{19}$$

Taking into account the stress and strain states for pure torsion and axial loading or bending under constant amplitudes and in the high cycle fatigue regime, the relationship between the coefficients a_L and b_L can be derived as [11]:

$$a_L = \frac{4 - \dfrac{\sigma_{aN}^2}{\tau_{aN}^2}}{1-\nu} \quad \text{and} \quad b_L = \frac{\sigma_{aN}^2}{\tau_{aN}^2(1+\nu)} \,. \tag{20}$$

Assuming that calculations will be performed in the high cycle fatigue regime under constant amplitude loading for bending and torsion, it is possible to calculate the amplitude of the equivalent stress directly from the energy parameter

$$\sigma_{aeq} = \sqrt{W_{aeq} 2E} \,. \tag{21}$$

3.4 Energy based criterion proposed by Socie

Socie [13] proposed the use of the uniaxial Smith-Watson-Topper parameter [14] in the critical plane, defined as the plane with maximum range of normal strains. Usually, to calculate the expected number of cycles, the energy-life characteristic is used

$$\sigma_{n,\max}\varepsilon_{an} = \sigma_a(N_f)\varepsilon_a(N_f) = \sigma'_f (2N_f)^b \left[\frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f (2N_f)^c \right], \tag{22}$$

obtained after modifications to the strain-life curve. However, in this work for the calculation of the Woehler curve the equivalent stress amplitude must be determined:

$$\sigma_{aeq} = \sqrt{E\sigma_{n,\max}\varepsilon_{an}} \,. \tag{23}$$

Comparing equations (21) and (23), it can be noted, that they are similar in form, but it should be remembered that the values included in the mentioned equations were determined in different planes. Also, if the experimentally obtained Woehler curves for bending and torsion show different slopes, the constants a_L and b_L become dependent on the level of the loading, i.e. number of cycles *N*.

3.5 Gough-Pollard ellipse

Gough and Pollard proposed a non-linear multiaxial fatigue criterion [16]. From investigations of in-phase combined bending and torsion on steel alloys they suggested a multiaxial fatigue strength criterion based on the applied stresses

$$\left(\frac{\sigma_{applied}}{\sigma_{a,pure,N}} \right)^{a_{GP}} + \left(\frac{\tau_{applied}}{\tau_{a,pure,N}} \right)^{b_{GP}} \le D_M \,. \tag{24}$$

By changing the values of parameters a_{GP} and b_{GP} different forms of the criterion can be obtained. However the best known is that according to Gough-Pollard $a_{GP} = b_{GP} = 2$ and $D_M = 1$, called also the Gough-Pollard hyperbola. Fatigue life assessment involves computing of the left side of the inequality (24) and if they are less then D_M then, for the given number of cycles, failure is not expected. For a notch the applied stresses can be calculated, particularly from the von Mises criterion, using only some selected components of the stress tensor

$$\left(\frac{\sqrt{\sigma_{ax}^2+\sigma_{ay}^2-\sigma_{ax}\sigma_{ay}}}{\sqrt{\sigma_{ax,N,pure}^2+\sigma_{ay,N,pure}^2-\sigma_{ax,N,pure}\sigma_{ay,N,pure}}}\right)^2+\left(\frac{\tau_{axy}}{\tau_{axy,N,pure}}\right)^2\leq D_M\,. \tag{25}$$

To take into account the variation of principal stress direction (out-of-phase multiaxial loading), the value of parameter D_M can be suitably changed. According to the recommendation [17], where the Gough-Pollard proposal for fatigue life determination of welded joints was used, the value of D_M must be lowered from 1.0 to 0.5. The presented method is not invariant, which means that any changes of the position of coordinate system XYZ influences the fatigue assessment. A big disadvantage of this method is that, during the calculation, the X axis must be parallel to the vector of torsion.

4 FATIGUE LIFE ASSESSMENT UNDER MULTIAXIAL STRESS STATES

In order to compare the selected multiaxial fatigue assessment criteria, calculated Woehler curves were outlined and compared with experimental ones. For this purpose, five algorithms for computing the stress amplitude according to the theoretical backgrounds of chosen criteria were worked out. Equivalent stress amplitudes for four different numbers of cycles: $2{\cdot}10^4$, $1{\cdot}10^5$, $4{\cdot}10^5$, and $4{\cdot}10^6$ were determined. All parameters included in the considered criteria were computed according to the material data for a given number of cycles from pure axial loading or bending and pure torsion. For notched specimens the multiaxial stress state was established according to the Neuber theory.

In the Dang Van method the parameters a_{DV} and b_{DV} were computed twice – using the experimental data from unnotched and notched specimens according to equations (11) and (12). The fatigue life assessment with the equivalent effective shear hypothesis was done using fatigue data obtained only from notched specimens according to equation (16). Because of the differences between stress gradients under bending and torsion, for calculation purposes, the slope of the torsion Woehler curve can be taken from the bending Woehler curve. In this case the knee point of the SN-curve was assumed at $N_k = 4{\cdot}10^5$ cycles and the slopes were $k_t = k_b$ =7.7 for Simbürger data and $k_t = k_b$ =4.6 for Atzori et al. data.

For the criterion proposed by Socie, the critical plane position determined by maximum normal strain must be known. This was identified by searching all available positions for maximum value of normal strain during one loading cycle. The critical plane position in the criterion proposed by Lagoda is described by the vector **n** normal to the plane and the vector **s**, which lies in the plane. For fixing the position of the two vectors, the maximum of stress in the direction of the vector **s** must be found by searching all possible positions of the vectors. An analytical formula for equivalent stress amplitude can-not be derived using the proposition of Gough-Pollard and the value was determined using an optimization procedure by changing the inequality sign in (25) to equality.

5 DISCUSSION AND EVALUATION OF THE RESULTS

The main conclusions of the present work can be summarised as follow:

- From the calculation results it can be concluded that the Effective Equivalent Stress Hypothesis gave the best results, Figs. 1 and 2. The calculated Woehler curves obtained are located on the safe side near to those obtained experimentally for all of the examined cases.
- On analysing the results obtained by the Dang Van criterion, Figs. 3 and 4, a tendency to overestimation of the fatigue life by out-of-phase loading and underestimation by in-phase loading can be noted. Also, better results were obtained according to the Dang Van line generated from data of notched specimens. This was expected, because these material data include the influence of stress gradients on the fatigue behaviour.
- With energy based methods offered by Łagoda and by Socie, acceptable results were not obtained. This was probably due to the fact that, for the calculations, local stress amplitudes were used without elastic-plastic analysis. These methods are recommended to be used together with strain-life curves, which describe the material properties better at significant plastic strains. With the Gough-Pollard criterion, conservative Woehler curves were determined. In this case, a higher value of the D_M parameter than 0.5 could be taken for evaluation of the fatigue life in out-of-phase loading.
- It can be noted that the received results for the Ck45 steel are much better than for the C40 steel. However the same tendencies for fatigue life assessment by the considered methods can be observed. This may be a result of the different failure criteria, crack initiation for Ck45 and total rupture for C40.
- The EESH method can be specified as the most reliable among the tested hypotheses. The Dang Van, Lagoda and Socie propositions are independent of the position of the coordinates system where only the stress tensor for fatigue life calculation is required. Therefore, the criteria proposed by Dang Van and Socie are often used in connection with FEM analysis. However, as they overestimate the fatigue life of ductile steels, they are not reliable.

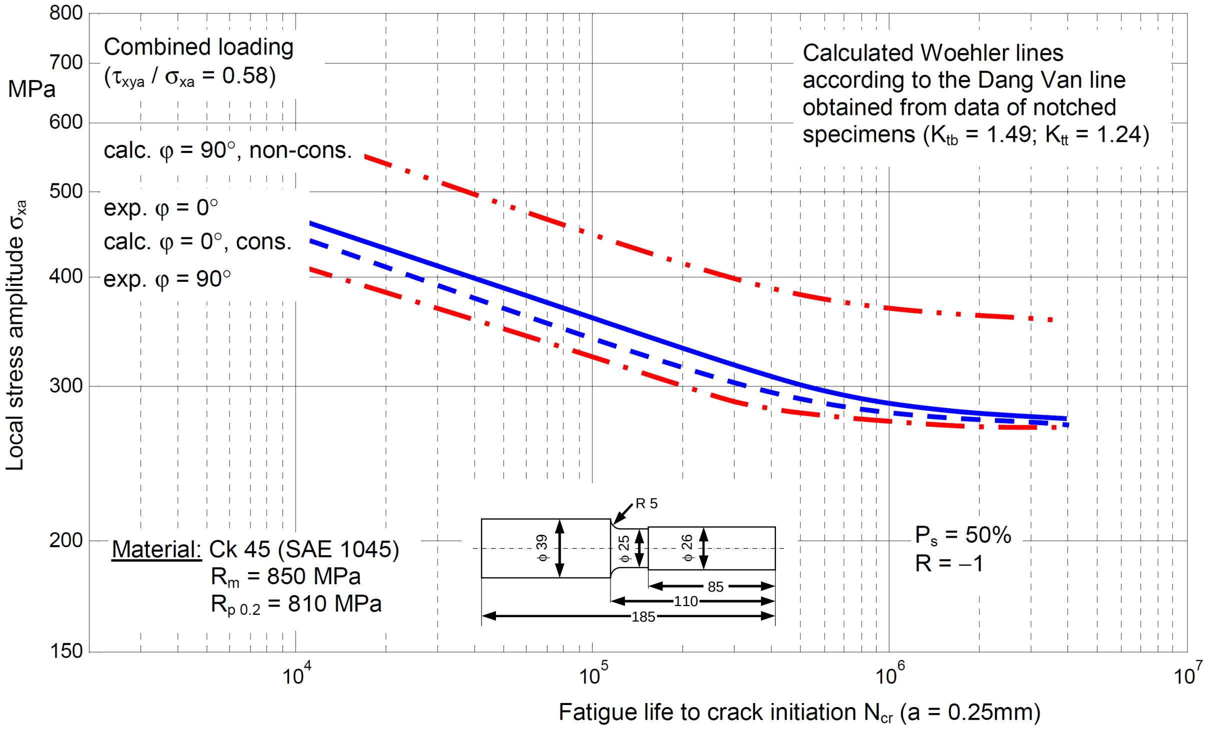

Figure 1: Evaluation of multiaxial stress states with the Dang Van hypothesis for notched specimens under combined bending and torsion. The Dang Van line obtained from data of notched specimens (K_{tb} = 1.49; K_{tt} = 1.24). Fatigue tests performed by Simbürger [3]

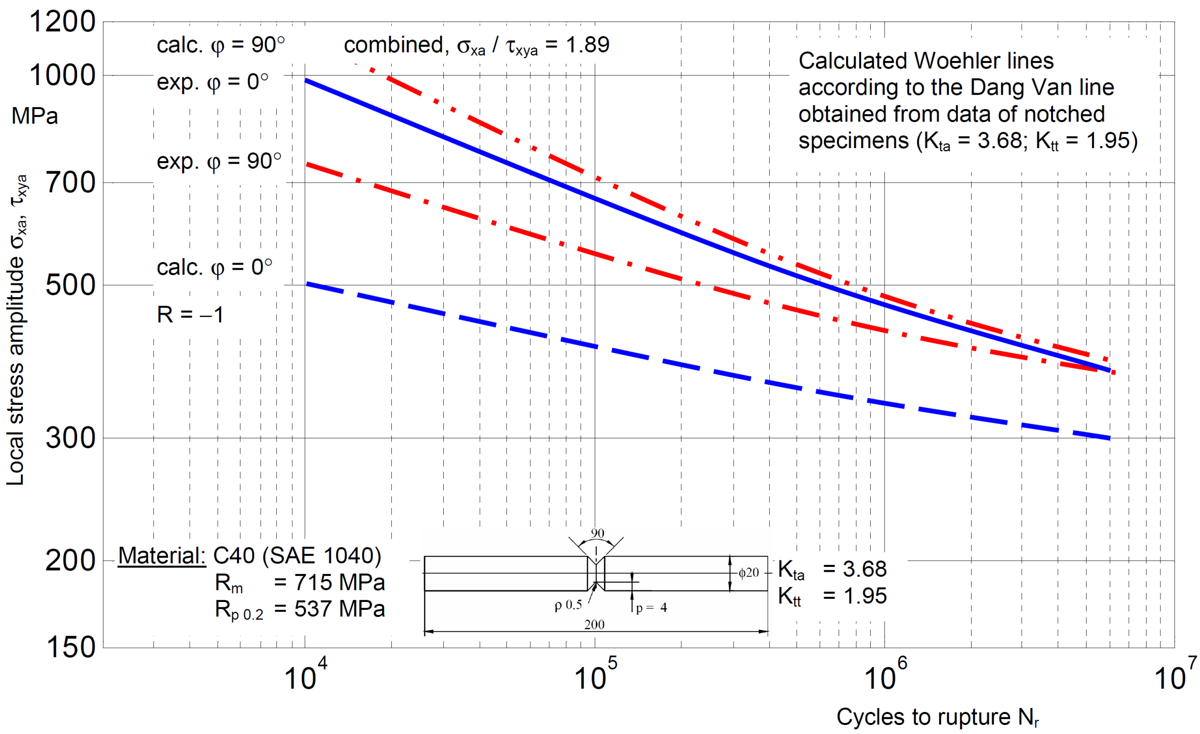

Figure 2: Evaluation of multiaxial stress states with the Dang Van hypothesis for notched specimens under combined bending and torsion. The Dang Van line obtained from data of notched specimens (K_{ta} = 3.68; K_{tt} = 1.95). Fatigue tests performed by Atzori et al. [4]

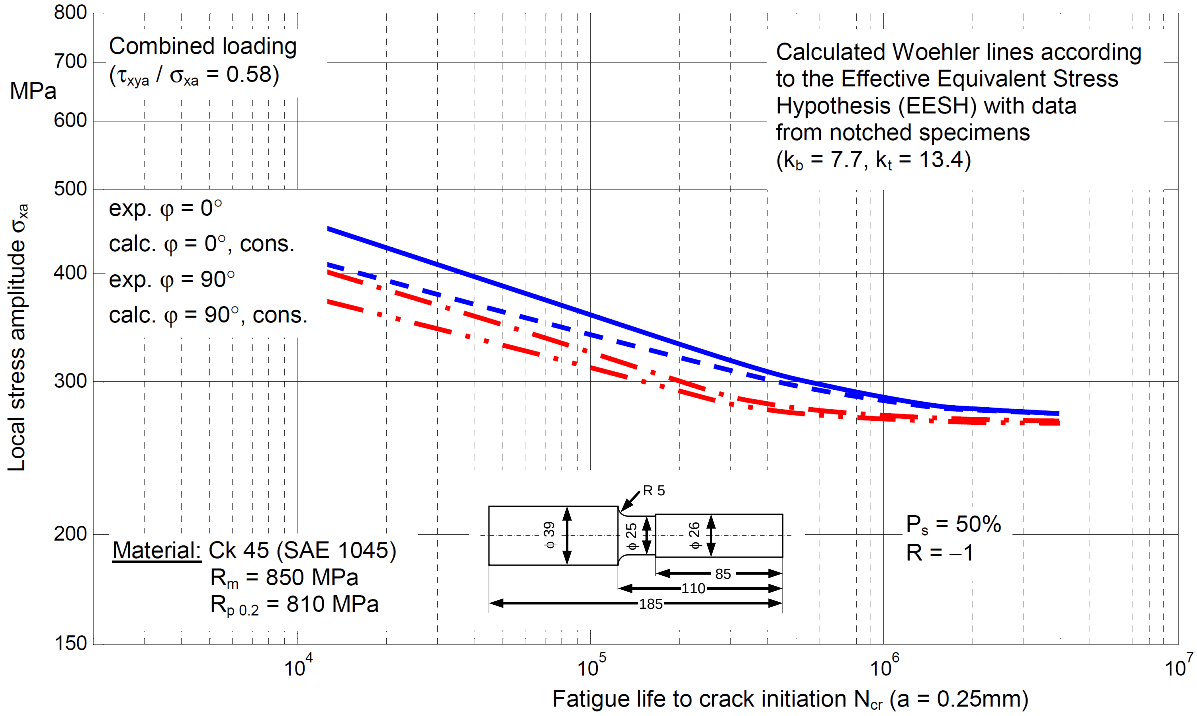

Figure 3: Evaluation of multiaxial stress states with the Hypothesis of Effective Equivalent Stress (EESH) for notched specimens under combined bending and torsion. Fatigue tests performed by Simbürger [3]

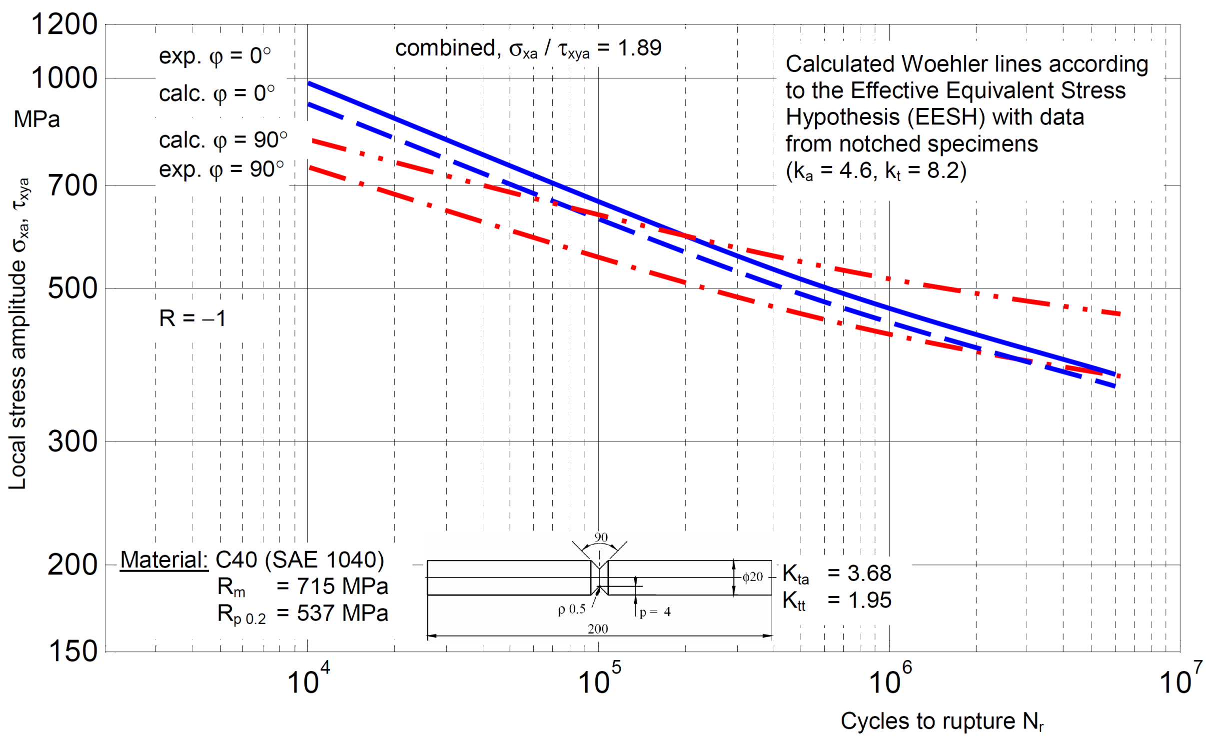

Figure 4: Evaluation of multiaxial stress states with the Hypothesis of Effective Equivalent Stress (EESH) for notched specimens under combined bending and torsion. Fatigue tests performed by Atzori et al. [4]

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